

A nonsingular rotating black hole

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Abstract The spacetime singularities in classical general relativity are inevitable, as predicated by the celebrated singularity theorems. However, it is a general belief that singularities do not exist in Nature and that they are the limitations of the general relativity. In the absence of a well-defined quantum gravity, models of regular black holes have been studied. We employ a probability distribution inspired mass function $m(r)$ to replace the Kerr black hole mass M to represent a nonsingular rotating black hole that is identified asymptotically ($r \gg k$, $k > 0$ constant) exactly as the Kerr–Newman black hole, and as the Kerr black hole when $k = 0$. The radiating counterpart renders a nonsingular generalization of Carmeli’s spacetime as well as Vaidya’s spacetime, in the appropriate limits. The exponential correction factor changing the geometry of the classical black hole to remove the curvature singularity can also be motivated by quantum arguments. The regular rotating spacetime can also be understood as a black hole of general relativity coupled to nonlinear electrodynamics.

The celebrated theorems of Penrose and Hawking [1] state that under some circumstances singularities are inevitable in general relativity. For the Kerr solution these singularities have the shape of a ring, this case is timelike. The Kerr metric [2] is undoubtedly the most remarkable exact solution in the Einstein theory of general relativity, which represents the prototypical black hole that can arise from gravitational collapse, which contains an event horizon [3]. Thanks to the no-hair theorem, the vacuum region outside a stationary black hole has a Kerr geometry. It is believed that spacetime singularities do not exist in Nature; they are a creation of general relativity. It turns out that we need to ask to what extent a singularity in general relativity could be adequately explained by some other theory, say, quantum gravity. However, we are yet afar from a specific theory of quantum gravity. So a suitable course of action is to understand the inside of a black hole and resolve its singularity by carrying out research of

classical black holes, with regular (nonsingular) properties, where spacetime singularities can be avoided in the presence of horizons. Thus, the regular black holes are solutions that have horizons and, contrary to classical black holes which have singularities at the origin, their metrics as well as their curvature invariants are regular everywhere [4]. This can be motivated by quantum arguments of Sakharov [5] and Gliner [6], who proposed that spacetime in the highly dense central region of a black hole and should be de Sitter-like for $r \simeq 0$, which was later explored and refined by Mukhanov et al. [7–10]. This indicates an unlimited increase of spacetime curvature during a collapse process, which may halt, if quantum fluctuations dominate the process. This puts an upper bound on the value of curvature and compels the formation of a central core.

Bardeen [11] realized the idea of a central matter core, by proposing the first regular black hole, replacing the singularity by a regular de Sitter core, which is a solution of the Einstein equations coupled to an electromagnetic field, yielding an alteration of the Reissner–Nordström metric. However, the physical source associated to a Bardeen solution was clarified much later by Ayon-Beato and Garcia [12]. The exact self-consistent solutions for the regular black hole for the dynamics of gravity coupled to nonlinear electrodynamics had also been obtained later [13–15], which also share most properties of the Bardeen’s black hole. Subsequently, there has been intense activity in the investigation of regular black holes as in [4, 16–20], and more recently in [21–24], but most of these solutions are more or less based on Bardeen’s proposal. However, non-rotating black holes cannot be tested by astrophysical observations, as the black hole spin plays a critical and key role in any astrophysical process. This prompted a generalization of these regular solutions to the axially symmetric case or to the Kerr-like solution [25–29], via the Newman–Janis algorithm [30] and by other, similar techniques [31–33]. It is also demonstrated that these rotating regular solutions can act as particle accelerator [34, 35]. The

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algorithm also allowed one to generate a noncommutative inspired rotating black hole with a regular de Sitter toroidal region [36]. However, these rotating regular solutions go over to the Kerr solution in the appropriate limits, but not to the Kerr–Newman solution.

This letter searches for a new class of three-parameter, stationary, axisymmetric metric that describes regular (non-singular) rotating black holes. The metric depends on the mass (M) and spin (a) as well a free parameter (k) that measures the potential deviation from the Kerr solution [2] and also generalizes the Kerr–Newman solution [30], which in Boyer–Lindquist coordinates reads

$$ds^2 = - \left(1 - \frac{2Mre^{-k/r}}{\Sigma} \right) dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 - \frac{4aMre^{-k/r}}{\Sigma} \sin^2 \theta d\theta d\phi + \left[r^2 + a^2 + \frac{2Mra^2e^{-k/r}}{\Sigma} \sin^2 \theta \right] \sin^2 \theta d\phi^2, \quad (1)$$

with $\Sigma = r^2 + a^2 \cos^2 \theta$, $\Delta = r^2 + a^2 - 2Mre^{-k/r}$, and M , a , and k are three parameters, which will be assumed to be positive. The metric (1) includes the Kerr solution as the special case if the deviation parameter $k = 0$ and the Schwarzschild solution for $k = a = 0$. In that case $M = 0$; the metric (1) actually is nothing more than the Minkowski spacetime expressed in spheroidal coordinates. When only $a = 0$, the rotating regular metrics (1) transform into

$$- \left(1 - \frac{2Me^{-k/r}}{r} \right) dt^2 + \left(1 - \frac{2Me^{-k/r}}{r} \right)^{-1} dr^2 + r^2 d\Omega^2 \quad (2)$$

with $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$. It is a spherically symmetrical regular metric [21, 23, 24], which is a modification of the Schwarzschild solution. It is easy to see that by employing the Newman–Janis algorithm [30] to static spherical regular solution (2), one obtains the rotating regular spacetime (1). The properties of metric (2) including the thermodynamics have also been analyzed [21, 24]. Interestingly, the exponential convergence factor is used in the formulation of the quantum gravity, which is finite to all orders in the Planck length [37]. Further, the inclusion of such quantum gravity effects makes other flat space quantum field theories similarly finite. Further, a finite quantum gravity theory can be used to resolve the cosmological constant problem [38]. Thus, the rotating regular metric (1) is the same as the Kerr black hole, but the mass (M) of the Kerr black hole is changed to $m(r)$,

$$m(r) = \frac{\sigma(r)}{\sigma_\infty} M, \quad (3)$$

where the function $\sigma(r) \geq 0$ and $\sigma'(r) < 0$ for $r \geq 0$, and $\sigma(r)/r \rightarrow 0$ as $r \rightarrow 0$, and $\sigma(\infty) = \sigma(r \rightarrow \infty)$ denotes

the normalization factor. Here, $\sigma(r) = \exp(-k/r)$ so that $\sigma(\infty) = 1$.

Note that the metric (1) asymptotically ($r \gg k$) behaves as a rotating counterpart of the Reissner–Nordström solution or the Kerr–Newman solution [30], i.e.,

$$g_{tt} = 1 - \frac{(2Mr - q^2)}{\Sigma} + \mathcal{O}(k^2/r^2), \\ \Delta = r^2 + a^2 - 2Mr + q^2 + \mathcal{O}(k^2/r^2).$$

This happens when the charge q and mass M are related with the parameter k via $q^2 = 2Mk$. Thus, the solution (1) is stationary, axisymmetric with Killing field $(\frac{\partial}{\partial t})^a$ and $(\frac{\partial}{\partial \phi})^a$, and all known stationary black holes are encompassed by the three-parameter family solutions, and it also generalizes the Kerr–Newman solution [30]. It is not difficult to find numerically a range of M and k for which the solution (1) is a black hole, in addition to being regular everywhere. Henceforth, for definiteness we shall call the solution (1) the regular (nonsingular) rotating black hole.

We approach the regularity problem of the solution by studying the behavior of the invariant $R = R_{ab}R^{ab}$ (R_{ab} is the Ricci tensor) and the Kretschmann invariant $K = R_{abcd}R^{abcd}$ (R_{abcd} is the Riemann tensor). It may be pointed out that the solution (1) is regular if the curvature invariants are well behaved [1, 13–15, 25, 29] (1); they read

$$R = \frac{2k^2 M^2 e^{-\frac{2k}{r}}}{r^6 \Sigma^4} \left(\Sigma^2 k^2 - 4r^3 \Sigma k + 8r^6 \right), \quad (4) \\ K = \frac{4M^2 e^{-\frac{2k}{r}}}{r^6 \Sigma^6} \left(\Sigma^4 k^4 - 8r^3 \Sigma^3 k^3 + Ak^2 + Bk + C \right),$$

where A , B , and C are functions of r and θ , given by

$$A = -24r^4 \Sigma (-r^4 + a^4 \cos^4 \theta), \\ B = -24r^5 (r^6 + a^6 \cos^6 \theta - 5r^2 a^2 \cos^2 \theta \Sigma), \\ C = 12r^6 (r^6 - a^6 \cos^6 \theta) - 180r^8 a^2 \cos^2 \theta (r^2 - a^2 \cos^2 \theta).$$

These invariants, for $M \neq 0$, are regular everywhere, including at $\Sigma = 0$, where it is noteworthy that they vanish (cf. Fig. 1).

We have also examined the other Ricci and Weyl invariant for the rotating metric (1) [39–42]; they can be obtained with Mathematica or Maple. It will be necessary to introduce the trace-free Ricci tensor defined by $S_b^a = R_b^a - \delta_b^a R/4$, where R_b^a is the Ricci tensor [39–42]. The non-zero Ricci invariant for the rotating metric (1) reads

$$RS = R_a^a = \frac{2k^2 M e^{-k/r}}{r^3 \Sigma}, \\ R1 = \frac{1}{4} S_a^b S_b^a = \frac{k^2 M^2 e^{-2k/r} (k \Sigma - 4r^3)^2}{r^6 \Sigma^4}, \\ R3 = \frac{1}{16} S_a^b S_b^c S_c^d S_d^a = \frac{1}{64} \frac{k^4 M^4 e^{-4k/r} (k \Sigma - 4r^3)^4}{r^{12} \Sigma^8}. \quad (5)$$

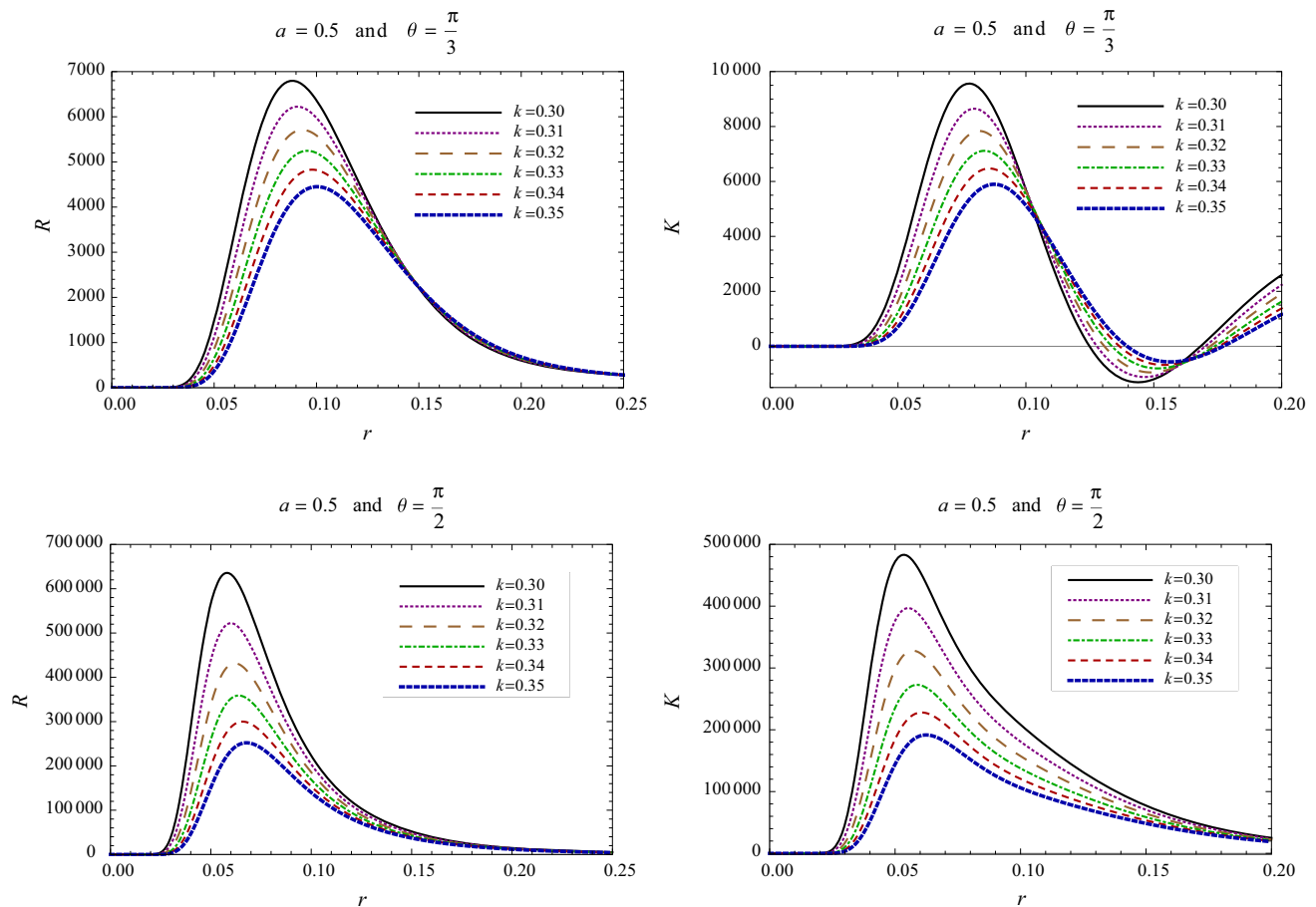


Fig. 1 Plots showing the regular behavior of the Ricci scalar and Kretschmann scalar vs. radius for the different values of parameter k (with $M = 1$)

Not surprisingly they are also regular everywhere. The Weyl invariant WII reads

$$WII = \frac{1}{8} = C_{ab}^{*cd} C_{cd}^{ab} = \frac{2aM^2 \cos \theta e^{-2k/r} \alpha \beta}{r^4 \Sigma^6} \quad (6)$$

with

$$\begin{aligned} \alpha &= k^2 a^4 \cos^4 \theta + 2r^2 a^2 (-9r^2 - 3rk + k^2) \cos^2 \theta \\ &\quad + r^4 (k^2 + 6r^2 - 6rk), \\ \beta &= a^2 (r + k) \cos^2 \theta + r^2 (-3r + k). \end{aligned} \quad (7)$$

Here C_{abcd} is the Weyl (conformal) tensor, and C_{abcd}^* its tensor dual. We shall not report the analytic form of the other Weyl invariants which are also obtained and found to be regular everywhere. Thus, the exponential factor $e^{-k/r}$ removes the curvature singularity of the Kerr black hole.

In addition, the solution (1) is singular at the points where $\Sigma \neq 0$, and $\Delta = 0$, and it is a coordinate singularity; this surface is called the event horizon (EH). The numerical analysis of the transcendental equation $\Delta = 0$ reveals that it is possible to find non-vanishing values of the parameters a

and k for which Δ has a minimum, and it admits two positive roots r_{\pm} . It turns out that $r = r_{\pm}$ are coordinate singularities of the same nature as the singularity at $r = 2M$ in the Schwarzschild spacetime, the metric can be smoothly extended across $r = r_+$, with $r = r_+$ being a smooth null hypersurface, and the simplest possible extension could be rewriting solution (1) in Eddington–Finkelstein coordinates, as shown below (cf. Eq. (14)). It turns out that, for a given a , there exists a critical value of k , of k_c^{EH} , and of r , r_c^{EH} , such that $\Delta = 0$ has a double root, which corresponds to a regular extremal black hole with degenerate horizons ($r_-^{\text{EH}} = r_+^{\text{EH}} = r_c^{\text{EH}}$). When $k < k_c^{\text{EH}}$, $\Delta = 0$ has two simple zeros and has no zeros for $k > k_c^{\text{EH}}$ (cf. Fig. 2). These two cases correspond, respectively, to a regular non-extremal black hole with a Cauchy horizon and an EH, and a regular spacetime. It is worthwhile to mention that the critical values of k_c^{EH} and r_c^{EH} are a dependent, e.g., for $a = 0.3, 0.5$, respectively, $k_c^{\text{EH}} = 0.63, 0.48$ and $r_c^{\text{EH}} = 0.82, 0.89$ (cf. Fig. 2). Indeed, the k_c^{EH} decreases with the increase in a ; on the other hand, the radius r_c^{EH} increases with an increase in a .

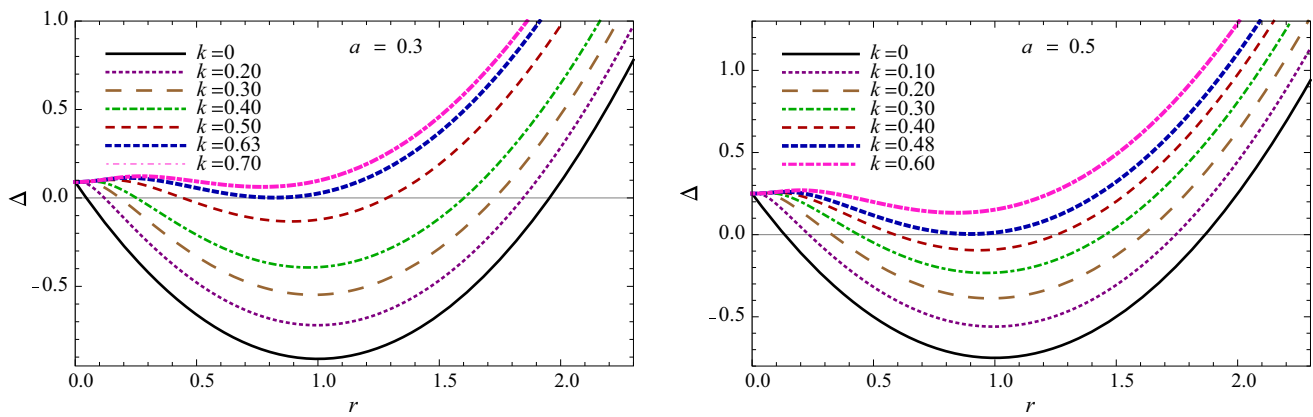


Fig. 2 Plots showing the behavior of Δ vs. radius for different values of parameter k (with $M = 1$); the case $k = 0$ corresponds to the Kerr black hole

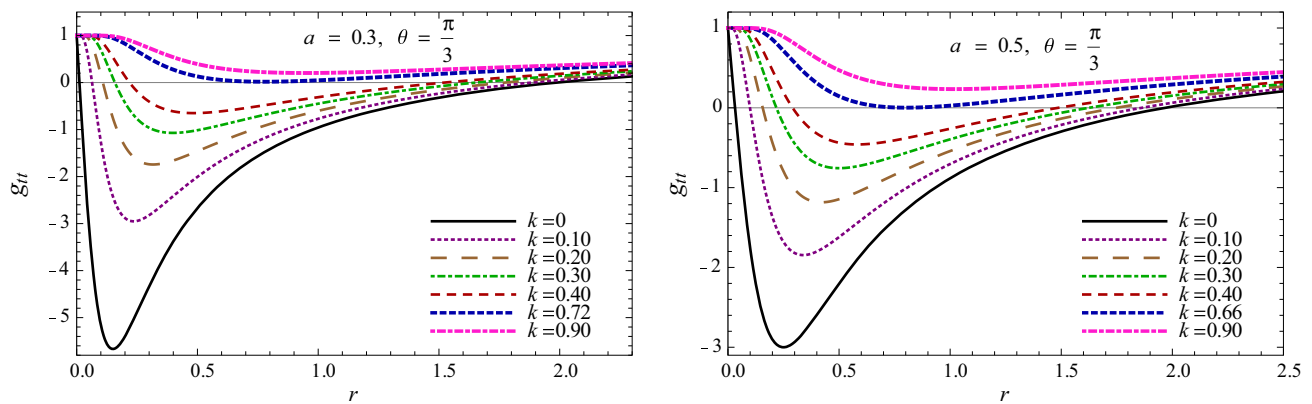


Fig. 3 Plots showing the behavior of $-g_{tt}$ vs. radius for the different values of the parameter k (with $M = 1$); the case $k = 0$ corresponds to the Kerr black hole

The timelike Killing vector $\xi^a = (\frac{\partial}{\partial t})^a$ of the solution has norm

$$\xi^a \xi_a = g_{tt} = - \left(1 - \frac{2Mre^{-k/r}}{\Sigma} \right), \quad (8)$$

and it becomes positive in the region where $r^2 + a^2 \cos^2 \theta - 2Me^{-k/r} < 0$. This Killing vector is null at the stationary limit surface (SLS), whose locations are, for different k , depicted in Fig. 3. The analysis of the zeros of $g_{tt} = 0$, for a given value of a and θ , disseminate a critical parameter k_c^{SLS} such that $g_{tt} = 0$ (e.g., $k_c^{\text{SLS}} = 0.72$, and 0.66 , respectively, for $a = 0.3$ and 0.5) has no roots if $k > k_c^{\text{SLS}}$, a double root at $k = k_c^{\text{SLS}}$, and two simple zeros if $k < k_c^{\text{SLS}}$ (cf. Fig. 3). Also, like in the case of EH, k_c^{SLS} and r_c^{SLS} have similar behavior with a for a given θ , as shown in Fig. 3. Interestingly, the radii, of EH and SLS for the solution decreases when compared to the analogous Kerr case ($k = 0$). Notice that for $\theta = 0, \pi$, the SLS and EH coincides. On the other hand, outside this symmetry, they do not (cf. Table 1) as in the usual

Kerr/Kerr–Newman. The region between $r_+^{\text{EH}} < r < r_+^{\text{SLS}}$ is called the *ergosphere*, where the asymptotic time translation Killing field $\xi^a = (\frac{\partial}{\partial t})^a$ becomes spacelike and an observer follows an orbit of ξ^a . The shape of the ergosphere, therefore, depends on the spin a and parameter k . It came as a great surprise when Penrose [46] suggested that energy can be extracted from a black hole with an ergosphere. On the other hand, the Penrose process [46] relies on the presence of an *ergosphere*, which for the solution (1) grows with the increase of parameter k as well with spin a as demonstrated in Table 1. This in turn is likely to have impact on the energy extraction, which is being investigated separately. The vacuum state is obtained by letting the horizon's size go to zero or by making the black hole disappear; this amounts to $r \rightarrow \infty$. One thus concludes that the solution is asymptotically flat as the metric components approaches those of the Minkowski spacetime in spheroidal coordinates.

In order to further analyze the matter associated with the metric (1), we use an orthonormal basis in which the energy–momentum tensor is diagonal [25, 29, 43],

Table 1 Radius of EHs, SLSs and $\delta^a = r_{\text{SLS}}^+ - r_{\text{EH}}^+$ for different values of the parameter k (with $M = 1$ and $\theta = \pi/3$)

$a = 0.3$				$a = 0.5$		
k	r_{EH}^+	r_{SLS}^+	$\delta^{0.3}$	r_{EH}^+	r_{SLS}^+	$\delta^{0.5}$
0	1.95394	1.98869	0.03475	1.86603	1.96825	0.10222
0.1	1.84577	1.88471	0.03894	1.74540	1.86184	0.11644
0.2	1.72956	1.77409	0.04453	1.61141	1.74802	0.13661
0.3	1.60235	1.65479	0.05244	1.45583	1.62422	0.16839
0.4	1.45860	1.52336	0.06476	1.25489	1.48593	0.23104
0.45	1.37716	1.45129	0.07413	1.10409	1.40876	0.30467

$$e_{\mu}^{(a)} = \begin{pmatrix} \sqrt{\mp(g_{tt} - \Omega g_{t\phi})} & 0 & 0 & 0 \\ 0 & \sqrt{\pm g_{rr}} & 0 & 0 \\ 0 & 0 & \sqrt{g_{\theta\theta}} & 0 \\ g_{t\phi}/\sqrt{g_{\phi\phi}} & 0 & 0 & \sqrt{g_{\phi\phi}} \end{pmatrix}, \quad (9)$$

with $\Omega = g_{t\phi}/g_{\phi\phi}$. Clearly, the metric is regular at the center. The components of the energy–momentum tensor in the orthonormal frame read

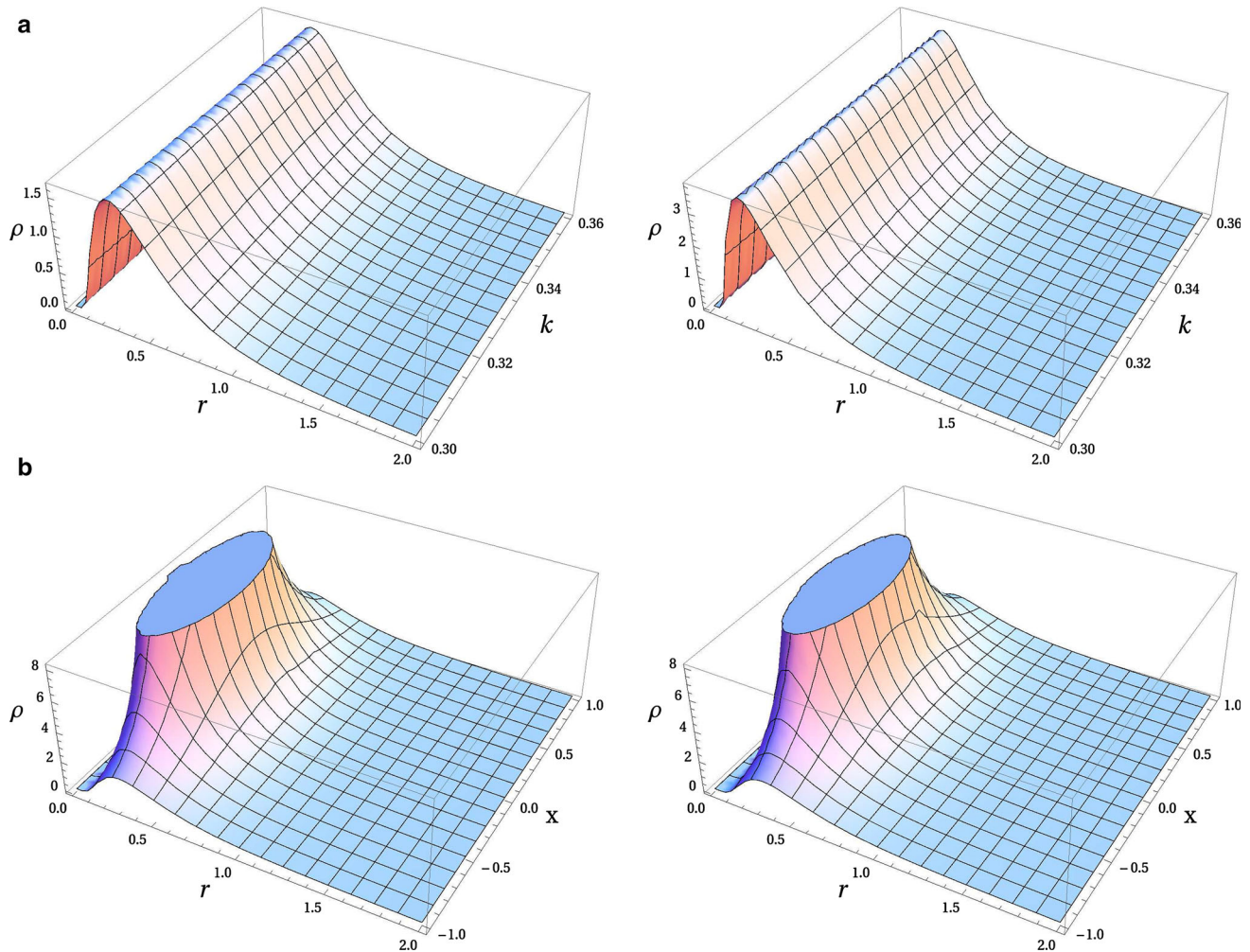
$$T^{(a)(b)} = e_{\mu}^{(a)} e_{\nu}^{(b)} G^{\mu\nu}.$$

Considering the line element (1), we can write the components of the respective energy–momentum tensor as

$$\begin{aligned} \rho &= \frac{2Mke^{-k/r}}{\Sigma^2} = -P_1, \\ P_2 &= -\frac{Mke^{-k/r}(k\Sigma - 2r^3)}{r^3\Sigma^2} = P_3. \end{aligned} \quad (10)$$

To check the weak energy condition, we can choose an appropriate orthonormal basis [25,29,43] in which the energy–momentum tensor reads

$$T^{(a)(b)} = \text{diag}(\rho, P_1, P_2, P_3). \quad (11)$$

**Fig. 4** Plots of ρ vs. radius. **a** Top for the different values of parameter k ($\theta = \pi/3$ (left) $\pi/4$ (right)) and **b** bottom for different values of $x = \cos \theta$ ($k = 0.3$ (left) $k = 0.4$ (right))

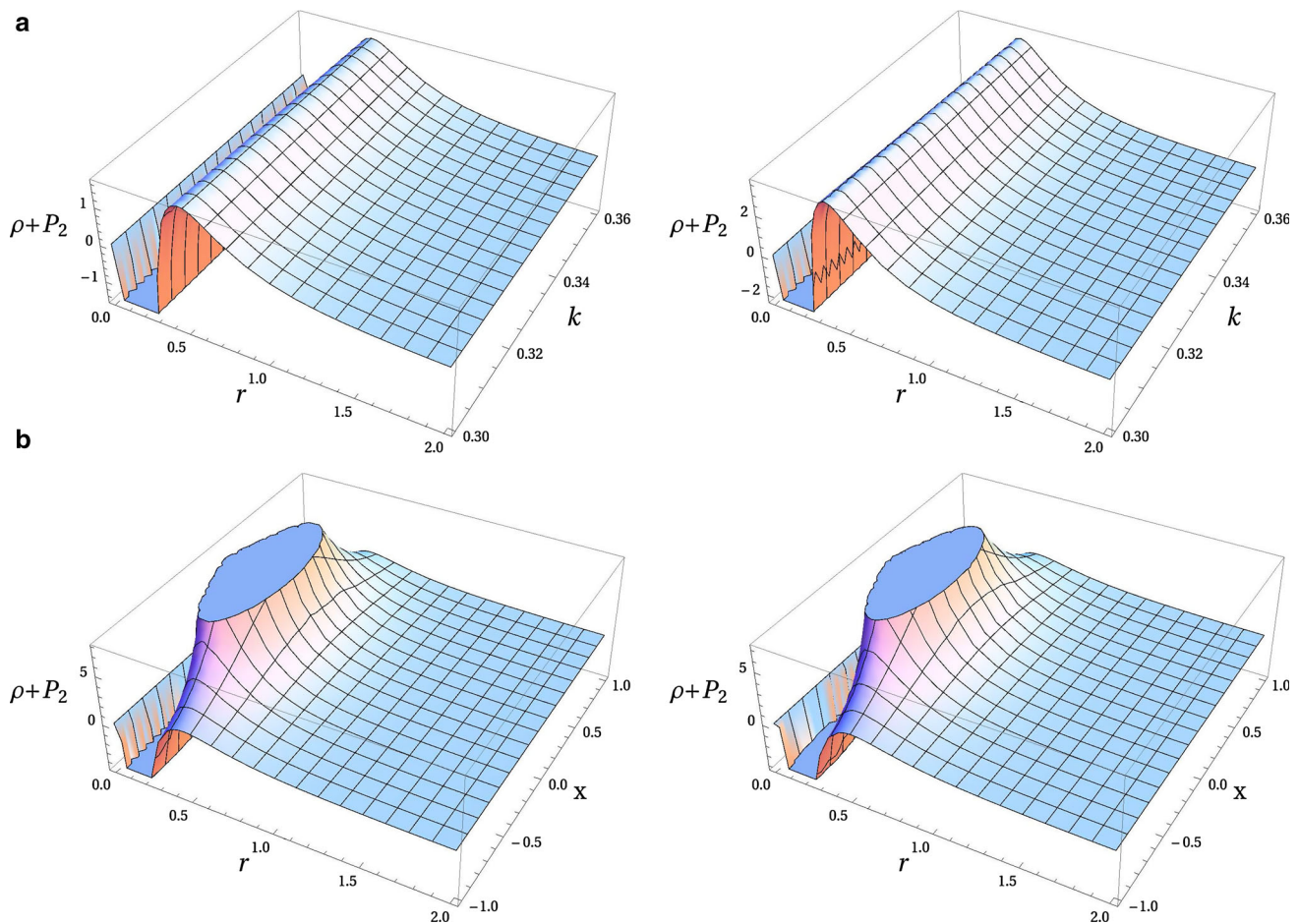


Fig. 5 Plots of $\rho + P_1 = \rho + P_2$ vs. radius. **a** Top for the different values of parameter k ($\theta = \pi/3$ (left) $\pi/4$ (right)) and **b** bottom for different values of $x = \cos \theta$ ($k = 0.3$ (left) $k = 0.4$ (right))

These stresses vanish when $k = 0$, and also for $M = 0$; they fall off rapidly at large r for $M, k \neq 0$; and for $r \gg k$ and $a = 0$, they are, to $O(k^2/r^2)$, exactly the stress-energy tensor of the Maxwell charge given by

$$T_{(b)}^{(a)} = T_b^a = \frac{q^2}{r^4} \text{diag}[-1, -1, 1, 1].$$

In this limit, the solution exactly takes the form of the Kerr–Newman solution. Further, the causal (horizon) structure of the solution (1) is similar to that of the Kerr solution, except that the scalar polynomial singularity of the Kerr solution, at center ($r = 0$), no more exists with regular behaviors of the scalars at the center, as shown in Fig. 1. Thus, the solution (1), which asymptotically behaves as Kerr–Newman, can be understood as a rotating regular black hole of general relativity coupled to a suitable *nonlinear electrodynamics*.

The weak energy condition requires $\rho \geq 0$ and $\rho + P_i \geq 0$ ($i = 1, 2, 3$) [1]. In fact, for our case one has

$$\rho + P_2 = \rho + P_3 = -\frac{Mke^{-k/r}(k\Sigma - 4r^3)}{r^3\Sigma^2}, \quad (12)$$

which shows that the violation of the weak energy condition for a regular black hole may not be prevented. The weak energy condition is not really satisfied, but the violation can be very small, depending on the value of k , as shown in Figs. 4 and 5.

Next, we add radiation by rewriting the static solution (1) in terms of the Eddington–Finkelstein coordinates (v, r, θ, ϕ) [44]:

$$v = t + \int \frac{r^2 + a^2}{\Delta}, \quad \bar{\phi} = \phi + \int \frac{a}{\Delta}, \quad (13)$$

and we allow the mass M and the parameter k to be a function of time v ; dropping the bar, we get

$$ds^2 = -\left(1 - \frac{2M(v)re^{-k(v)/r}}{\Sigma}\right)dv^2 + 2dvdr + \Sigma d\theta^2 - \frac{4aM(v)re^{-k(v)/r}}{\Sigma} \sin^2 \theta dv d\phi - 2a \sin^2 \theta dr d\phi$$

$$+ \left[r^2 + a^2 + \frac{2M(v)ra^2e^{-k(v)/r}}{\Sigma} \sin^2 \theta \right] \sin^2 \theta d\phi^2. \quad (14)$$

Again relating $M(v)$, $q(v)$ with $k(v)$ as done earlier, all stresses of the solution (14) have the same form as that of the solution (1), but (14) has some additional stresses corresponding to the energy–momentum tensor of ingoing null radiation [45]. The solution (14) describes the exterior of radiating objects, recovering the Carmeli solution (or rotating Vaidya solutions) [45] for $k = 0$, and the Vaidya solution [46] when $k = a = 0$. The radiating rotating solution (14) is a natural generalization of the stationary rotating solution (1), but it is Petrov type-II with a twisting, shear free, null congruence; the same as for a stationary rotating solution, which is of Petrov type D . Thus, the radiating solution (14) bears the same relation to the stationary solution (1) as does the Vaidya solution to the Schwarzschild solution.

To construct the said rotating regular black hole, we have used an exponential regularization factor, suggested by Brown [37], used in a quantum gravity that is finite to the Planck scale. The mass function $m(r)$ is also inspired by continuous probability distributions to replace the mass M of the Kerr black hole.

We have given an example of a rotating regular solution showing that (1) contains the Kerr metric as a special case when the deviation parameter, $k = 0$, and it also for $r \gg k$ behaves as Kerr–Newman and it is stationary, axisymmetric, asymptotically flat. It turns out that the rotating regular black hole metrics (1) can also be obtained via the widely used Newman–Janis algorithm [30]. It will be useful to further study the geometrical properties, causal structures, and thermodynamics of the black hole solution which is being investigated.

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